- 1. The ecliptic plane is not the fundamental plane for the perifocal coordinate system. (TRUE/FALSE)
- 2. Altering the longitude of the ascending node  $\Omega$  causes the orbital plane to rotate about the Earth's z-axis. (TRUE/FALSE)
- 3. The angular momentum vector  $\mathbf{h}$  and the third axis of the perifocal coordinate system  $\mathbf{W}$ , point along the same direction. (TRUE/FALSE)
- 4. The maximum velocity of a satellite in an elliptic orbit occurs at its periapsis. (TRUE/FALSE)
- 5. The true anomaly  $\nu$  is the angle between the eccentricity vector and the position vector. (TRUE/FALSE)
- 6. A satellite in orbit around the Earth has a constant specific angular momentum vector **h**. What two things does this imply about the orbit?
- 7. Give the symbol, name and a brief description of the six classical orbital elements. The description should include (but does not need to be limited to):
  - a. whether the element is a shape, orientation, or location factor.
  - b. what the element is measuring and, if it is an angle, what it is measured with respect to.
- 8. State Kepler's three laws.
- 9. Write Newton's law of universal gravitation.
- 10. Complete the table below with the proper value, range of the allowed values, or algebraic sign of each quantity.

 $\mathcal{E}$  (energy) e (eccentricity)

Circle

Ellipse

Parabola

Hyperbola

- 11. The space shuttle is in obit about the earth with a perigee altitude of 292 km and a period of 90.3 minutes. Determine the following:
  - a. semi-major axis,
  - b. apogee altitude,
  - c. specific mechanical energy,
  - d. eccentricity,
  - e. magnitude of the specific angular momentum, and
  - f. radial position and velocity when the true anomaly is 150°.
- 12. A satellite is injected into orbit with  $\mathbf{r} = -1.75\mathbf{j}$  DU and  $\mathbf{v} = 0.70\mathbf{k}$  DU/TU. Without detailed calculations, classify the orbit and determine as many of the classical orbital elements as possible.
- 13. Radar has detected a spaceborne object with  $\mathbf{r} = -1.04\mathbf{k}$  DU and  $\mathbf{v} = 1.2\mathbf{i}$  DU/TU. Determine the classical orbital elements, classify the orbit, and find the altitude of closest approach. Is it a missile, satellite, or space probe?

14	Given:
14.	GIVCII.

$$r_p = 1.75$$
 DU  
 $e = 0.25$   
 $i = 90^{\circ}$   
 $\Omega = 0^{\circ}$   
 $\omega = 170^{\circ}$   
 $\nu = 260^{\circ}$ 

## Determine:

- a.  $\mathbf{r}$  and  $\mathbf{v}$  in the perifocal plane,
- b.  $\mathbf{r}$  and  $\mathbf{v}$  in the geocentric-equatorial coordinate system, and
- c. use your answers in b. to calculate the orbital elements and verify your answers.
- 15. Choose the definition from those given below which best define the following terms and write the answer in the blank provided. There may or may not be more than one definition associated with each blank.

Apogee	
Periapsis	
Perihelion	
Inclination	
Argument of perigee	
Flight path angle	
Ecliptic	
True anomaly	
Eccentricity	
Semi-major axis	

- a. A constant defining the shape of a conic section
- b. 24 hours
- c. A constant defining the size of a conic section
- d. The angle between the eccentricity vector and the radius vector measured in the orbital plane
- e. 23 hrs 56 min 4 sec
- f. The point on a conic section closest to its primary focus
- g. The shortest distance from the center of the Moon to an orbiting body
- h. The farthest point from the center of the Earth to a geocentric orbit
- i. The farthest point from the center of the Moon to an elliptical lunar orbit
- j. The closest point from the center of the Earth to a geocentric orbit
- k. The farthest point from the center of the Sun to a heliocentric orbit
- 1. The closest point from the center of the Sun to a heliocentric orbit
- m. The angle between the Earth's polar axis and the normal to the plane of the Earth's orbit
- n. The plane of the Earth's orbit around the Sun
- o. The angle between the orbit plane normal and the normal to the equatorial plane
- p. The angle between the velocity vector of an orbiting body and the perpendicular to the radius vector measured in the orbital plane
- q. The angle between the ascending noed line and the radius vector at orbit periapsis measured in the orbital plane
- 16. The orbit of a body whose specific energy is zero is
  - a. hyperbolic
  - b. elliptical
  - c. circular
  - d. parabolic

17	. The orbit of a body whose eccentricity is zero is
	a. hyperbolic
	b. elliptical
	c. circular
	d. parabolic
18	. The velocity of a satellite in a parabolic orbit at an infinite distanced from the center of mass is a. infinite
	b. zero
	c. termed hyperbolic excess velocity
	d. undefined
19	. The flight path angle for a circular orbit is
	a. 0°
	b. 45°
	c. 90°
	d. none of the above
20	d. none of the above  The fact that two-body orbits are planar evolves from considering the conservation of

21. He was a meticulous observer of planetary motion, but a poor mathematician. a. Kepler

d. energy

a. mass

b. Copernicus

b. linear momentum c. angular momentum

- c. Brahe
- d. Aristotle

22. The fundamental plane for the perifocal coordinate system is

- a. the plane of the Earth's orbit around the Sun
- b. the Earth's equatorial plane
- c. the plane of the satellite's orbit
- d. none of the above

23. A body may escape from its attracting mass if its speed is greater than

- a.  $\frac{\mu}{r}$ b.  $2\sqrt{\frac{\mu}{r}}$ c.  $\sqrt{\frac{\mu}{r}}$ d.  $\sqrt{\frac{2\mu}{r}}$

24. It is desired to change the inclination of a satellite in an elliptical Earth orbit. The point where minimum  $\Delta v$  would be required for the task would be

- a. apoapsis
- b. periapsis
- c. the end of the semi-major axis
- d. none of the above

25. On an elliptical orbit, when the vehicle is at a true anomaly of 90°, the eccentric anomaly is

- a.  $> 90^{\circ}$
- b.  $< 90^{\circ}$

- c.  $= 90^{\circ}$
- 26. The period of a geostationary orbit is
  - a. 24 hours of mean solar time
  - b. 365 days
  - c. less than the period of a low circular orbit
  - d. 23 hr 56 min 04 sec of mean solar time
- 27. The eccentricity vector e
  - a. has the magnitude of the orbit eccentricity and points in the direction of the ascending node
  - b. has the magnitude of the angular momentum and points in the direction of periapsis
  - c. has the magnitude of the orbit eccentricity and points in the direction of the periapsis
  - d. none of the above

A satellite is orbiting the Earth with eccentricity 0.05 and a semi-major axis of 6900 km. Orbital inclination is  $30^{\circ}$  and the satellite was launched easterly. ( $\mu = 3.986 \times 10^5 \text{ km}^3/\text{s}^2$ )

- 28. The specific angular momentum of the satellite is
  - a.  $52,378 \text{ km}^2/\text{s}$
  - b.  $19,568 \text{ km}^2/\text{s}$
  - c.  $51,116 \text{ km}^2/\text{s}$
  - d.  $25,037 \text{ km}^2/\text{s}$
- 29. When the true anomaly of the satellite is 90°, its altitude is
  - a. 539.25 km
  - b. 877.00 km
  - c. 177.00 km
  - d. 504.75 km
- 30. The specific energy of the satellite is
  - a.  $28.884 \text{ km}^2/\text{s}^2$
  - b.  $2.8841 \text{ km}^2/\text{s}^2$
  - c.  $-28.884 \text{ km}^2/\text{s}^2$
  - d.  $-2.8841 \text{ km}^2/\text{s}^2$
- 31. When a satellite makes a flyby on the backside of a planet, its velocity with respect to the planet
  - a. increases
  - b. decreases
  - c. stays the same while its velocity with respect to the Sun
  - d. increases
  - e. decreases
  - f. stays the same
- 32. The maximum velocity for a satellite occurs when the true anomaly is 180°. (TRUE/FALSE)
- 33. Minimum radial distance for a satellite occurs when it achieves minimum velocity. (TRUE/FALSE)
- 34. A satellite whose energy is less than zero is able to escape the body which attracts it. (TRUE/FALSE)
- 35. A satellite on a hyperbolic trajectory has just enough velocity to escape the Earth; but when it does so, it has no velocity left over. (TRUE/FALSE)

- 36. The inertial velocity of a spacecraft increases when it performs a flyby behind a planet. (TRUE/FALSE)
- 37. The eccentric anomaly, true anomaly and mean anomaly of a satellite have the same value when the satellite is at apogee. (TRUE/FALSE)
- 38. Kepler discovered that planets move in elliptical orbits around the Sun, but was unaware of the reaon for this behavior. (TRUE/FALSE)
- 39. The angular momentum of a satellite in an elliptical orbit varies because the flight path angle varies. (TRUE/FALSE)
- 40. Hohmann transfers are used frequently because they are fast: the time required is small. (TRUE/FALSE)
- 41. A spacecraft is in a highly eccentric Earth orbit whose perigee is in the upper atmosphere. As time goes by . . .
- a. The orbit's eccentricity stays the same as the orbit shrinks and the spacecraft finally reenters. (TRUE/FALSE)
- b. The orbit's perigee stays the same until its apogee comes down to match it. The circular orbit then shrinks until reentry occurs. (TRUE/FALSE)
- c. The spacecraft gains energy from the heat of the upper atmosphere and stays up indefinitely. (TRUE/FALSE)
- 42. Give the name and symbol of the orbital elements which describe the orientation of an orbital plane around the earth.
- 43. Changing  $\underline{\phantom{a}}$  causes the orbital plane to rotate about the  $\mathbf W$  axis of the perifocal coordinate system.
- 44. The ecliptic plane is the fundamental plane for the \_\_\_\_\_\_coordinate system.
- 45. All six of the orbital elements remain constant throughout an orbit. (TRUE/FALSE)

Fill in the blanks with the appropriate historical figures.

46. used 's data to discover the trajector	ory of Mars.
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- 47. \_\_\_\_\_introduced the heliocentric model of the solar system which allowed him to explain the apparent retrograde motion of the planets.
- 48. Although \_\_\_\_\_\_described the behavior of bodies in motion, \_\_\_\_\_\_described the properties that caused those bodies to move.
- 49. The Space Shuttle places a satellite into an orbit whose perigee radius is 6478 km and whose apogee altitude ie 300 km. Calculate the satellite's semi-major axis, eccentricity, specific mechanical energy, period and the magnitude of the angular momentum. Also, calculate its radius, velocity and flight path angle when the true anomaly is 375°. Use 6378 km for the radius of the Earth.
- 50. An Earth satellite is in a circular orbit whose altitude is 150 km. One of the station-keeping thrusters on the satellite erroneously misfires, imparting an instantaneous  $\Delta v$  of 4 km/s to the spacecraft. What type of orbit is the satellite now in? Support your conclusion with numerical data.
- 51. An interplanetary probe is in a circular parking orbit with an altitude of 0.5 DU. How much  $\Delta v$  does the probe need in order to escape the Earth with a hyperbolic excess velocity of 1 DU/TU? Fill in the blanks with the **name and symbol** of the appropriate orbital elements.

52.	is the only orbital element that changes during a satellite's orbit.					
53.	,andare the orbital elements that describe the orien-					
	tation of a satellite's orbital plane around the Earth.					
54.	The ecliptic plane is the fundamental plane for the perifocal coordinate system. (TRUE/FALSE)					
55.	5. Altering the longitude of the ascending node $\Omega$ causes the orbital plane to rotate about the Earth's axis. (TRUE/FALSE)					
56.	The angular momentum vector ${\bf h}$ and the third axis of the perifocal coordinate system ${\bf W}$ point along the same direction. (TRUE/FALSE)					
57.	The true anomaly is the angle between the eccentricity vector and the position vector. $(TRUE/FALSE)$					
58.	Changingcauses the orbital plane to rotate about the line of nodes.					
59.	The fundamental plane of the heliocentric coordinate system is					
60.	List the constants of the motion for the two-body problem.					
61.	STS-6, launched 4 April 1983, was piloted by Colonel Karol Bobko, USAF. Its orbit had a perigee altitude of 284 km and a period of 90.3 min. Determine the following:  a. semi-major axis  b. apogee altitude  c. specific mechanical energy  d. eccentricity  e. magnitude of the specific angular momentum vector  f. radial position and velocity when the true anomaly is 300°					
62.	A satellite is in a circular orbit 300 km above the Earth's surface. To transfer the satellite to a different orbit, it is given an instantaneous $\Delta v$ of 1 km/s in the direction of motion. What is the semi-major axis, period and apogee radius of the resulting trajectory?					
63.	A satellite in in a circular orbit 300 km above the Earth's surface. To transfer the satellite to a different orbit, it is given an instantaneous $\Delta v$ of 4 km/s opposite the direction of motion. Describe the resultant orbit, with numerical data to support your conclusions.					
64.	A satellite in orbit about the Earth has a constant specific mechanical energy $\mathcal{E}$ . What does this imply about the radial distance and velocity of the satellite?					
65.	List the range of values that the specific mechanical energy $\mathcal{E}$ may have and what these values imply about $(i)$ the shape of the orbit and $(ii)$ the ability of the satellite to escape its attracting body.					
66.	An interplanetary spacecraft approaches a planet in a hyperbolic orbit with a velocity at infinity of $V_{\infty}$ . When the satellite reaches the periapsis point, a maneuver is performed to enter a circular orbit with radius $r_p$ about the planet. Show that the $\Delta v$ required for this maneuver is					
	$\Delta v = \sqrt{v_{\infty}^2 + \frac{2\mu}{r_p}} - \sqrt{\frac{\mu}{r_p}}.$					
67.	is the angle between periapsis and the position of the satellite.					

- 68. \_\_\_\_\_\_is a constant defining the shape of the orbit.
  69. \_\_\_\_\_is the tilt of the orbit away from the Earth's equatorial plane.
- 70. \_\_\_\_\_\_is the angle between the unit vector **I** and the point where the satellite crosses the equatorial plane in a northerly direction.
- 71. A solar/siderial (choose the correct answer) day is the time required for the Earth to rotate once on its axis realtive to the stars.
- 72. A space shuttle crew places a satellite in a circular orbit with an altitude of 300 km.
  - a. After several days, a malfunction occurs which causes the station-keeping thrusters to misfire. As the orbital analyst on watch, the mission commander comes to you and tells you that the NORAD tracking system has located the satellite and determined the position and velocity vectors:

$$\mathbf{r} = \mathbf{I} - 3\mathbf{J}$$
 DU  
 $\mathbf{v} = 0.5\mathbf{I} + 0.2\mathbf{J}$  DU/TU

The commander wants to know what the orbital elements are: calculate them. If some of the angles are undefined, explain why.

b. That pesky satellite has wandered off again, and now NORAD has given you the orbital elements for the satellite:

$$a = 7000 \quad \text{km}$$
 
$$e = 0.2$$
 
$$i = 5^{\circ}$$
 
$$\Omega = 30^{\circ}$$
 
$$\omega = 45^{\circ}$$

The mission commander wants to know the position vector and velocity vector (expressed in the perifocal coordinate system) when the satellite is at apogee. Calculate them.

c. If the following transformation between (P, Q, W) and (X, Y, Z) is known to be true

$$\begin{pmatrix} P \\ Q \\ W \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$$

then express  $\mathbf{r}$  and  $\mathbf{v}$  in the (X,Y,Z) coordinate system.

- 73. You are a member of the planning team for an unmanned mission to Mercury intended to place a high-resolution photographic and radar mapping satellite in polar orbit. The purpose of the satellite is to search for any topographic changes due to tectonic activity that may have occurred since the last mission to Mercury. Your team specifically is assigned the task of creating a flight plan that will take the spacecraft from launch to proper deployment in Mercury orbit. You have two days before your mission manager must brief the preliminary mission plan to the NASA Administrator and convince him to authorize the project.
  - a. The mission manager wants the spacecraft to arrive when Mercury is at aphelion. Calculate the  $\Delta v$  associated with a Hohmann transfer to Mercury aphelion and with a Hohmann transfer to Mercury perihelion to determine which is less. For this comparison, assume a circular solar orbit for the Earth and ignore gravitational escape for both Earth and Mercury.
  - b. Calculate the phase angle and transit time for aphelion arrival.

- c. Calculate the total  $\Delta v$  necessary to take the spacecraft from a 250-km low-earth parking orbit to a 65-km operational orbit around Mercury.
- d. Assuming the spacecraft masses 250 kg dry and carries propellant with a specific impulse of 330 sec, calculate the amount of propellant required.
- e. What changes if any need to be made to account for the fact that Mercury's orbit has 7° inclination?
- f. What should be the inclination of the low-earth parking orbit? Calculate the launch azimuth for Plesetsk, Commonwealth of Independent States (Russia). (Lat 68.10°)
- 74. A satellite in earth orbit has the following radius vectors:

$$\mathbf{r} = -3.28 \,\mathbf{i} + 0 \,\mathbf{k}$$
  
 $\mathbf{v} = -0.138 \,\mathbf{i} - 0.390 \,\mathbf{j} + 0.390 \,\mathbf{k}$ 

Find the orbital elements  $a, e, i, \Omega, \omega$  and  $\nu$ . If some of the elements are undefined, explain why.

- 75. An earth satellite is in polar orbit (a = 10,000 km, e = 0.2) with its perigee above the north pole. It has two receiving stations, one at the north pole and one at the south pole. The satellite is visible to the north pole station when it is above 30°N latitude, and visible to the south pole station when it is avove 30°S latitude. How much time (in seconds) during each orbit is the satellite visible to a receiving station?
- 76. As an orbital analyst working at NORAD, it is your job to determine what type of trajectory a piece of space debris has. Your boss hands you this information:

$$\mathbf{r} = -0.707 \,\mathbf{i} + 0.707 \,\mathbf{j} \quad \mathrm{DU}$$
$$\mathbf{v} = 0.5 \,\mathbf{j} \quad \mathrm{DU/TU}$$

and tells you to give him the classical orbital elements. What are  $a, e, i, \Omega, \omega$  and  $\nu$ ? If some of the quantities are undefined, explain why.

77. Now your boss hands you these orbital elements for an earth satellite,

$$a = 10,000$$
 km  
 $e = 0.2$   
 $i = 30^{\circ}$   
 $\Omega = 0^{\circ}$   
 $\omega = 45^{\circ}$ 

and tells you to find the equations for  $\mathbf{r}$  and  $\mathbf{v}$  in the perifocal coordinate system when the satellite reaches perigee. What are they?

78. A space station colony in a 1.25 AU orbit around the sun sustained life-threatening damage when a wayward spacecraft piloted by an EA362 flunkee collided with it. An an EA362 distinguished graduate, you are tasked by the US Space Force (USSF) to pilot a vehicle to rescue the surviving colonists. The USSF vehicle you have been given allows you to travel a trajectory with an eccentricity of 0.75. Your departure point will be from Earth, which makes the radius of perigee for your trajectory equal to 1 AU. You will time your departure so that you will rendezvous with the colony the <u>first</u> time your trajectory

reaches 1.25 AU. At the time of your departure, USSF has calculated that the colony has only 60 days of air left. Will you be able to reach the colony in time to rescue the survivors?

79. A satellite in Earth orbit has the following radius and velocity vectors:

$$r = 1.5 i - 4.5 j$$
 DU  
 $v = 0.5 i + 0.2 j$  DU/TU

Find the orbital elements  $a, e, i, \Omega, \omega$  and  $\nu$ . If some of the elements are undefined, explain why.

- 80. A radiation sensing satellite is in an elliptical polar orbit with a perigee <u>altitude</u> of 300 km and an apogee radius of  $5.0R_E$ . This orbit causes the satellite to pass through the Van Allen radiation belt regions of highest density which are between  $1.5R_E$  and  $4.0_E$ . Draw a sketch of the orbit and calculate the following:
  - a. semi-major axis a, in km,
  - b. eccentricity e.
  - c. the amount of time in seconds, that the satellite spends in the radiation belts.
- 81. An earth satellite is known to have the following radius and velocity vectors:

$$\mathbf{r} = \mathbf{i} + \mathbf{j} + \mathbf{k}$$
 DU  $\mathbf{v} = \mathbf{k}$  DU/TU

What are  $\mathbf{r}$  and  $\mathbf{v}$  expressed in the perifocal coordinate system when the true anomaly is 45°?

82. As an orbital analyst working at NORAD, it is your job to determine what type of trajectory a piece of space debris has. Your boss hands you this information:

$$\mathbf{r} = -0.2 \,\mathbf{i} - 0.5 \,\mathbf{j}$$
 DU  
 $\mathbf{v} = 0.3 \,\mathbf{i} - 0.4 \,\mathbf{j}$  DU/TU

and tells you to give him the classical orbital elements. What are  $a, e, i, \Omega, \omega$  and  $\nu$ ? If some of the quantities are undefined, explain why.

83. Determine **r** and **v** in the perifocal coordinate system for a satellite with the following orbital elements:

$$a=8000$$
 km  
 $e=0.15$   
 $i=45^{\circ}$   
 $\Omega=60^{\circ}$   
 $\omega=90^{\circ}$   
 $\nu=30^{\circ}$ 

- 84. A satellite is in an elliptical orbit with e = 0.2 and a = 9000 km. It is in polar orbit with perigee directly over the North Pole. How much time during each orbit does the satellite spend north of 30 deg S latitude?
- 85. A satelite is in an elliptical orbit with an eccentricity of e = 0.2 and a semi-major axis of 10,000 km. On a single graph, plot the radial distance r and the true anomaly  $\nu$  over the span of one period.
- 86. A satellite is in an elliptical orbit with e = 0.1 and a = 8000 km. It is in a polar orbit with the <u>apogee</u> directly over the North Pole.

- a. How long does it take for the satellite to go from perigee to a point where the radial distance is 8400 km?
- b. How much time during each orbit does the satellite spend north of 45° N latitude?
- 87. A lunar module (LM) lifts off from the lunar surface and flies directly into a Hohmann transfer ellipse whose perigee <u>altitude</u> (the burnout point) is 30 km. The transfer ellipse will allow the LM to rendezvous with the Apollo Command Module (CM) which is in a 250 km (altitude) circular orbit. (The mass of the Moon is 0.01213 times the mass of the Earth, and the radius of the Moon is 1740 km.)
  - a. What is the burnout speed of the LM in km/s?
  - b. What  $\Delta v$  is required to rendezvous with the CM in km/s?
  - c. What is the time of flight (TOF) from burnout to rendezvous for the LM?
  - d. To assure a quick rendezvous, it is desirable that the LM and the CM arrive at the rendezvous point together (or at least close). Where must the CM be in relation to the LM at burnout? Cite your answer as a time differential (in seconds) of the CM ahead or behind of the LM burnout point.
- 88. Answer the following questions by inspection of the table below.
  - a. Object \_\_\_\_\_\_ is on a parabolic orbit.
  - b. Object \_\_\_\_\_\_ is on a circular orbit.
  - c. Object \_\_\_\_\_ is on a hyperbolic orbit.
  - d. Object \_\_\_\_\_\_ is at periapsis.
  - e. Object \_\_\_\_\_\_ is in retrograde orbit.
  - f. Object \_\_\_\_\_\_ is in prograde orbit.
  - g. Object \_\_\_\_\_ has a specific angular momentum vector lying entirely within the I-J plane.
  - h. Object \_\_\_\_\_ has zero total specific energy.
  - i. Object \_\_\_\_\_\_ is at apoapsis.
  - j. Object departs from a point  $\mathbf{r} = 1\mathbf{K}$  DU with  $\mathbf{v} = \sqrt{2}\mathbf{I} + \sqrt{2}\mathbf{J}$  DU/TU.

Object	a	e	i	Ω	$\omega$	$ u_0$
A	$2 \mathrm{DU}$	0	0 °	Undefined	Undefined	Undefined
В	$\infty$	1	93°	90°	-90°	0°
С	-0.5 DU	3	90°	$225^{\circ}$	90°	0°

- 89. A spacecraft has semi-major axis a = -20,000 km. What kind of orbit is this? Find the total energy of the spacecraft. Derive an equation for the excess velocity of the spacecraft.
- 90. An orbital transfer vehicle (OTV) is scheduled for a maintenance mission on a spacecraft in geostationary orbit (altitude=35,800 km). The OTV is launched from the space shuttle which is in a coplanar orbit with an altitude of 322 km. After a positive collinear  $\Delta v$  is imparted, the OTV departs the Shuttle orbit with a geocentric velocity of 10.5 km/s. It needs to intercept and enter the orbit of the geostationary spacecraft.
  - a. Find the  $\Delta v$  required to put the OTV on the transfer trajectory from the Shuttle orbit.
  - b. Find the velocity, true anomaly and flight path angle upon arrival at the geostationary orbit.
  - c. Calculate the  $\Delta v$  required to enter the geostationary orbit.
  - d. Calculate the lead angel (the angle the geostationary spacecraft needs to be ahead of the Shuttle at the time of OTV launch) necessary for a successful intercept.
- 91. The Galileo spacecraft was originally supposed to utilize a heliocentric Hohmann transfer to Jupiter. Assume that Jupiter is in a circular 5.2 AU orbit which is coplanar with Earth's 1 AU circular orbit.

- a. Determine the following orbital properties of the transfer orbit:  $\mathcal{E}_t$ ,  $e_t$ ,  $a_t$ ,  $r_{tp}$ ,  $v_{tp}$ ,  $v_{ta}$  and TOF (transfer time between planets).
- b. Draw a sketch of the transfer orbit and label all the associated distances and velocities, including those of the planets. Also, indicate (and support with calculations) the position of Jupiter at the time of Galileo's launch from Earth.
- c. Determine the  $\Delta v$  needed to escape Earth from the 300 km altitude circular Shuttle parking orbit and "patch" into the Hohmann transfer orbit to Jupiter. Calculate the following quantities:

  - 1.  $e_{esc}$ : eccentricity of escape trajectory 2.  $v_{\infty}^E$ : outbound hyperbolic excess velocity
  - 3.  $\delta/2$ : half turning angle
  - 4.  $\mathcal{E}_{esc}$ : energy of escape trajectory
  - 5.  $v_{hyp}$ : hyperbolic velocity at  $r_{park}$
  - 6.  $v_{park}$ : parking orbit velocity
  - 7.  $\Delta v$
- d. Draw a fairly accurate sketch of the escape. Lable all the distances, velocities and angles. Also show where the  $\Delta v$  must be applied.
- e. Sketch the hyperbolic passage that would result for a back-side passage of Jupiter with a 10,000 km CPA (closest point of approach). Determine:
  - 1.  $v_{\infty}^{J}$ : inbound hyperbolic excess velocity
  - 2.  $e_{hyp}$ : eccentricity of hyperbolic trajectory
  - 3.  $v_{hyp}$ : hyperbolic velocity at CPA
  - 4.  $\delta$ : turning angle
  - 5.  $v_{inertial}$ : inertial velocity of the probe after the flyby.
- 92. An earth satelllite has the following orbital elements:

$$a = 8500 \quad \mathrm{km}$$
 
$$e = 0.2$$
 
$$i = 5 \quad \mathrm{deg}$$
 
$$\Omega = 30 \quad \mathrm{deg}$$
 
$$\omega = 45 \quad \mathrm{deg}$$

- a. What is its position vector  $\mathbf{r}$  and velocity vector  $\mathbf{v}$  (expressed in the perifocal coordinate system) when the satellite is at apogee?
- b. In order to transform the perifocal system used in part (a.) into an (x,y,z) coordinate system, the system is rotated about the Q axis through an angle of 30 degrees. Express  $\mathbf{r}$  and  $\mathbf{v}$  from part (a.) in the (x,y,z) coordinate system.